



## Quantification of Uncertainty Related to PMP Parameters

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**Abstract**--Probable Maximum Precipitation (PMP) calculation methodology involves a significant amount of judgment and uncertainty. This is a direct result of using observed extreme storm events to represent a theoretical upper limit of rainfall that likely has never been observed. The assumption is made that the observed extreme storm events represent the same storm environment that would occur during a PMP event. In most cases, hydrologists and engineers are provided PMP depths for a given area size and duration without necessarily having the appropriate context regarding the uncertainty in the development process and how that could affect the design and implementation decisions. Applied Weather Associates (AWA) has been developing PMP estimates for nearly 20 years and has developed many updates to the PMP estimation methodology and implemented best practices of how to best apply the results. Continuing this trend of innovation and improvement, AWA has built on previous work to provide a detailed uncertainty bounds associated with major components of PMP development. The uncertainty analysis quantified the range of PMP depths associated with the best estimate, allowing the hydrologist/engineer to make more informed decisions on how to best implement the results. This presentation will provide examples of the uncertainty analysis and compare the results to site-specific PMP depths at two large basins.

### I. INTRODUCTION

A detailed uncertainty analysis of the parameters important for PMP development was performed by Applied Weather Associates (AWA). This utilized the storm-based deterministically derived PMP depths developed for two basins extending from northern Minnesota through northwestern Ontario. The methodology used to quantify uncertainty in PMP is based on methods discussed in [1]. AWA considered seven main sources of uncertainty, as compared to five in [1], that may affect the site-specific PMP estimates and whose uncertainties could be readily described as simple proportions of the original PMP estimate. The site-specific methods to calculate PMP for the basins were estimated using methods described in this report. For this uncertainty analysis, the 24-hour, basin average depths for each basin were used for comparisons in this analysis. The final basin average 24-hour PMP values are shown in Table 1.

TABLE I  
Basin 1 and Basin 2, 24-hour site-specific PMP estimates.

Basin	Area (km <sup>2</sup> )	24-hour PMP (mm)
Basin 1	67,491	170
Basin 2	71,675	189

### II. METHODS

AWA considered seven parameters for evaluation. Each of these are important for PMP calculation and include meteorological judgment with a range of possible outcomes based on understanding and data availability. The original Micovic paper utilize five parameters. AWA's analysis also included the range of uncertainty associated with rainfall analyses and judgment applied as part of the envelopment process. The final parameters considered include the following:

Storm Representative Dew Point Selection ( $T_{drep}$ )  
Transposition Limits ( $T_{rlim}$ )  
100-year Dew Point Climatology ( $T_{d100yr}$ )  
SPAS and Precipitation Analysis Accuracy ( $Ppt_{all}$ )  
Envelopment Process ( $E_p$ )  
Storm Efficiency ( $E_f$ )  
Storm Centering ( $S_c$ )

The likelihood functions depicting the seven PMP parameters were derived based on judgment and experience with the uncertainty characteristics for each of the factors. AWA has developed and calculated each of these parameters as part of our numerous PMP studies. We therefore have a good understanding of the range of variations of each parameter and the effect the each has individually and in aggregate on PMP depths.

As a result of the conservative nature of the procedures and policies for estimation of PMP, the majority of the likelihood functions are more restrictive in the direction of smaller PMP estimates and less restrictive to the possibility of larger PMP estimates [1].

The process for creating the shape of the likelihood functions starts with setting the upper and lower bounds which determines the range for a given factor (e.g., +/-10%). This range is based on meteorological judgment and known variation of a given parameter on final PMP depth estimation. The relative likelihoods of the smallest and largest values are then considered along with any central tendency for a given factor [1]. An equally likely likelihood function is used when there are no discernible differences across the range for a given factor. Empirical cumulative distribution functions (CDFs) were created by first integrating the area under the likelihood functions and then rescaling to an area of unity. The final PDFs are shown in Figure 1.

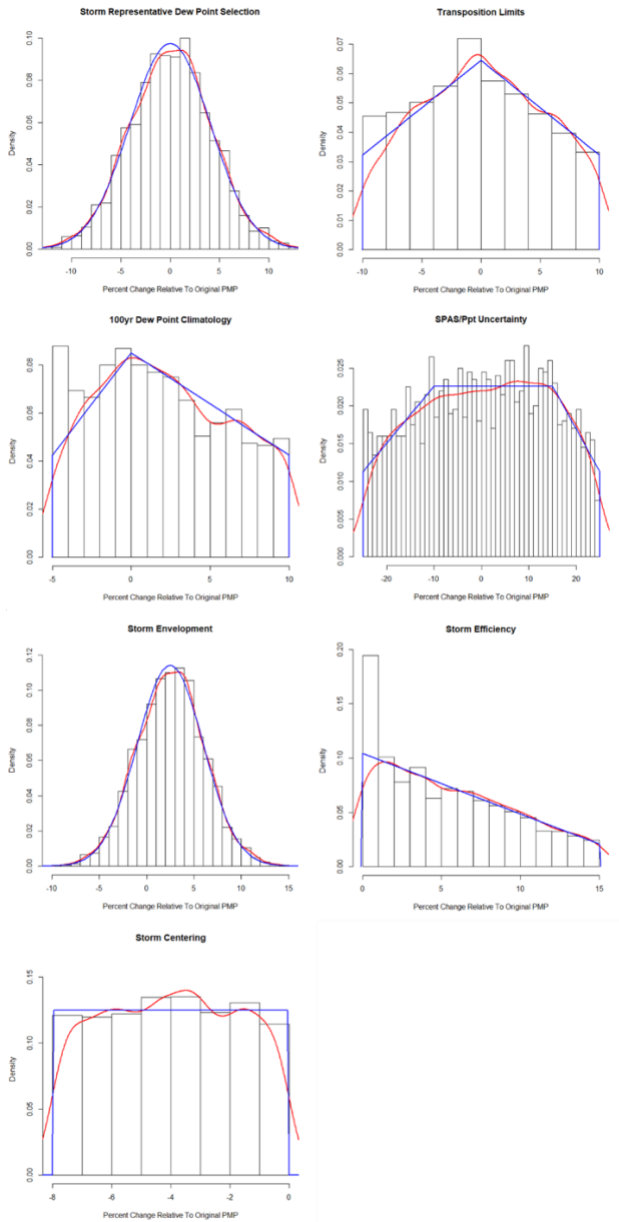


Figure 1. Likelihood functions for the effect of uncertainties relative to the 24-hour PMP estimates (blue line). The sampled values from the likelihood function (red line) and resulting sample histogram.

The uncertainty analysis was conducted using Monte Carlo simulation methods by considering the contribution from each of the seven sources of uncertainty described above. Latin-hypercube sampling methods were used to assemble 2000 sample sets comprised of combinations of the seven sources of uncertainty. The method used to account for uncertainties followed conventional approach for PMP estimation:

$$PMP_{24h,e,w} = PMP_{e,w} * Td_{rep} * T\eta_{lim} * Td_{100yr} * Ppt_{all} * E_p * E_f * S_c \quad (1)$$

where re  $PMP_{24h,e,w}$  is the 24-hour PMP value (mm) derived through Monte-Carlo uncertainty analysis analyses;  $PMP_{e,w}$  is the 24-hour PMP for the Basin 1 (e) and Basin 2 (w) basins;  $Td_{rep}$  is the Storm Representative Dew Point Selection probability distribution sample;  $T\eta_{lim}$  is Transposition Limits probability distribution sample;  $Td_{100yr}$  is the 100-year Dew Point Climatology probability distribution sample;  $Ppt_{all}$  is the SPAS and Precipitation Accuracy probability distribution sample;  $E_p$  is the Envelopment Process probability distribution sample; is the  $E_f$

is the Storm Efficiency probability distribution sample; and  $S_c$  is the Storm Centering probability distribution sample.

Each source of uncertainty was considered independent of the other sources. The combined effect of the seven uncertainty sources was considered to operate as described in Equation 1, with each component being a linear multiple of the original PMP estimates (Table 1). The sensitivity of the PMP estimate to the various factors can be inferred from the range and magnitude of the likelihood functions shown in Figure 1.

### III. RESULTS

The likelihood functions are expressed as a percentage of the PMP estimate and estimation of PMP is a multiplicative process. The resultant distribution of PMP estimates is depicted by the histogram in Figure 2 for Basin 1 and Figure 3 for Basin 2 and values for selected percentiles are shown in Table 2. Table 2 shows that the mean value for 24-hour PMP is 180 mm (106% of the original estimate) when uncertainties are considered for Basin 1 and is 201 mm (106% of the original estimate) when uncertainties are considered for Basin 2.

TABLE II  
Summary statistics for the 24-hour PMP uncertainty analysis for Basin 1 and Basin 2

Uncertainty Percentile	Basin 1		Basin 2	
	24-hour PMP (mm)	Ratio to Original PMP	24-hour PMP (mm)	Ratio to Original PMP
5%	135	0.79	150	0.79
10%	143	0.84	158	0.84
20%	154	0.91	171	0.90
50%	180	1.06	201	1.06
80%	210	1.24	234	1.24
90%	225	1.32	251	1.33
95%	237	1.39	264	1.40

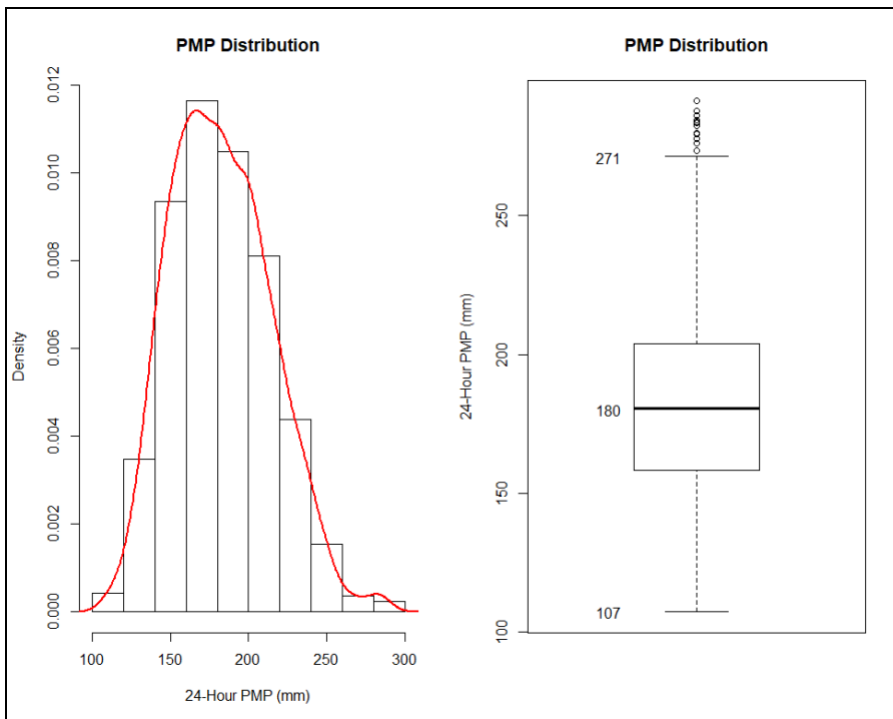


Figure 2. Histogram and boxplot of estimates of the 24-hour Basin 1 PMP based on 2000 simulations for seven sources of uncertainty.

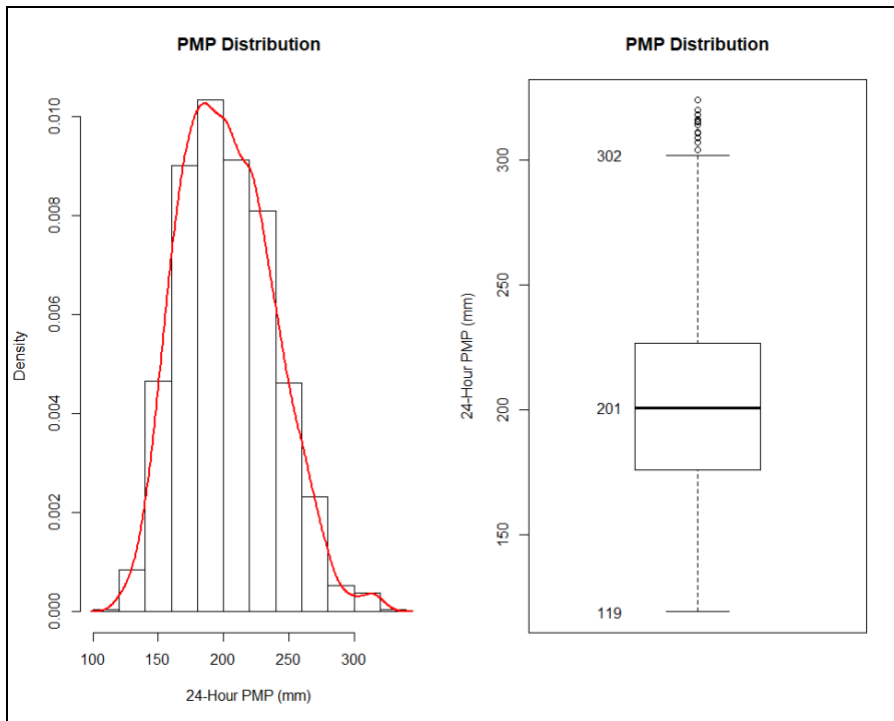


Figure 3. Histogram and boxplot of estimates of the 24-hour Basin 2 PMP based on 2000 simulations for seven sources of uncertainty.

For sensitivity purposes, Storm Efficiency ( $E_f$ ) and Storm Centering ( $S_c$ ) likelihood functions were removed. This was done because these two parameters are considered well quantified in this study given the assumption that at least one of the storms analyzed achieved maximum storm efficiency and because various storm centering locations were utilized for PMF development. The resultant distribution of PMP estimates without  $E_f$  and  $S_c$  are depicted by the histogram in Figure 4 for Basin 1 and Figure 5 for Basin 2 and values for selected percentiles are shown in Table 3. This shows that removal of these two parameters has very little effect on the overall range of values, where the mean value for 24-hour PMP is 178 mm (105% of the original estimate) when uncertainties are considered for the Basin 1 and is 198 mm (105% of the original estimate) when uncertainties are considered for the Basin 2.

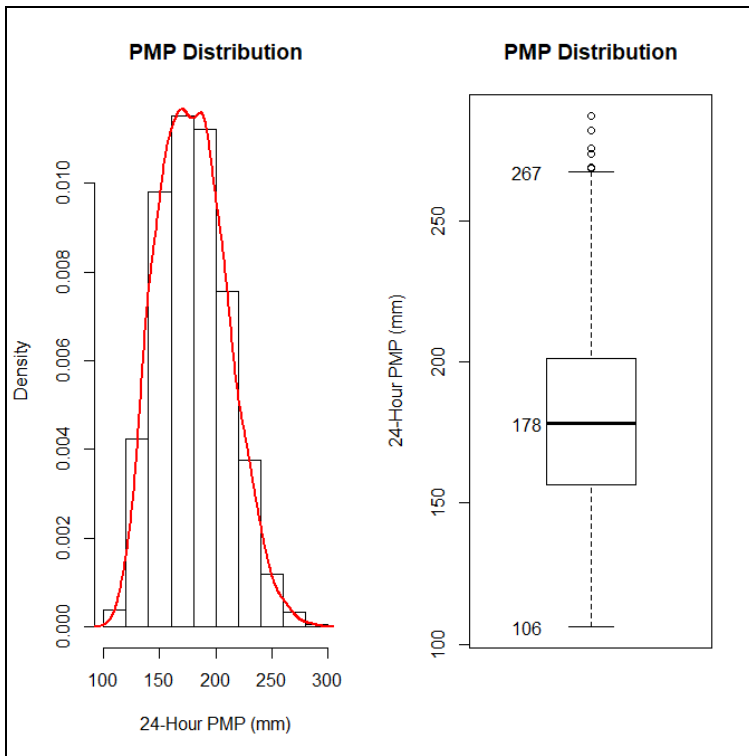


Figure 4. Histogram and boxplot of estimates of the 24-hour Basin 1 PMP based on 2000 simulations for five sources of uncertainty (without  $E_r$  and  $S_c$  uncertainties).

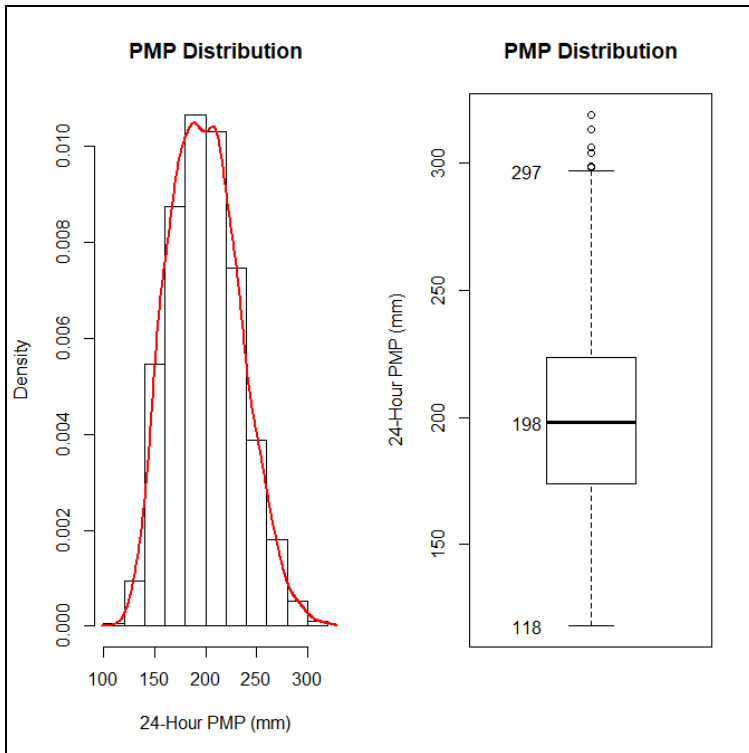


Figure 5. Histogram and boxplot of estimates of the 24-hour Basin 2 PMP based on 2000 simulations for five sources of uncertainty (without  $E_r$  and  $S_c$  uncertainties).

TABLE III

Summary statistics for the 24-hour PMP uncertainty analysis for Basin 1 and Basin 2 without  $E_r$  and  $S_c$  uncertainties.

Uncertainty Percentile	Basin 1		Basin 2	
	24-hour PMP (mm)	Ratio to Original PMP	24-hour PMP (mm)	Ratio to Original PMP
5%	134	0.79	149	0.79
10%	141	0.83	157	0.83
20%	153	0.90	170	0.90
50%	178	1.05	198	1.05
80%	206	1.21	229	1.21
90%	221	1.30	246	1.30
95%	233	1.37	259	1.37

#### IV. CONCLUSION

The result of this uncertainty analysis demonstrates that the PMP estimates developed using standard deterministic, storm-based procedures produced depths are reasonable and acceptable, as they fall in the middle of the overall range. This also confirms many of the assumptions and judgments applied during the PMP develop process. In particular, the assumption that the storm-based deterministic processes are valid in producing PMP estimates that can be used for design of high-hazard structures is appropriate.

#### V. REFERENCES

1. Micovic, Z., Schaefer, M.G., and G.H. Taylor, 2015: Uncertainty Analysis For Probable Maximum Precipitation Estimates. J. Hydrology., 521, 360–373.

#### VI. AUTHOR BIOGRAPHIES

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Mr. Kappel is President and Chief Meteorologist of Applied Weather Associates (AWA). Mr. Kappel received an AA from Skagit Valley College in 1992, a BS in Physical Science from Colorado Mesa University in 1998 and a Broadcast Meteorology degree from Mississippi State University in 2001. He served as an on-air meteorologist for 10 years at various television stations across the country prior to joining AWA in 2003. Mr. Kappel is the technical lead in all aspects related to PMP development and project management at AWA. Mr. Kappel has been the project manager for more 100's of PMP studies globally while working extensively in the development, analysis, and publication various meteorological variables. Mr. Kappel has also been involved in several forensic meteorology cases, meteorological input parameters development for use in hydrologic model calibration/validation, and rain-on-snow melt calculations. As part of the PMP and extreme storm analysis work, Mr. Kappel has investigated the effects of climate change on PMP and precipitation production covering numerous emission and future projections.

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Dr. Hultstrand is a Senior HydroMeteorologist at Applied Weather Associates with 19+ years of experience in meteorology, hydrology, and hydrometeorology. He has been involved in all aspects of site-specific, statewide and regional probable maximum precipitation (PMP) development, extreme storm analyses, regional precipitation frequency analyses, uncertainty analysis, a developer of the Storm Precipitation Analysis System (SPAS),

quantified annual exceedance probability (AEP) of PMP, developed rainfall areal reduction (ARF) factors, performed climate change analysis, and performed forensic meteorological investigations. Dr. Hultstrand has been a guest instructor at Colorado State University and the University of Colorado at Boulder, an Executive Program Committee Member for the Data Analysis and Modeling group for the National Hydrologic Warning Council, has published and peer reviewed numerous articles in the academic, scientific, and private communities, and acted as an independent peer-review member for the Journal of Hydrometeorology, Journal of Hydrologic Engineering, and the US Geological Survey Scientific Investigation Reports (SIR).