# TEMPORAL DISTIRUBTION OF PMP RAINFALL AS A FUNCTION OF AREA SIZE 

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## Introduction

As part of the process of deriving Probable Maximum Precipitation (PMP) values that provide rainfall amounts for use in computing the Probable Maximum Flood (PMF) it is important to remember that the methods used to derive PMP and the processes which utilize those values hydrologically adhere to the caveat of being "physically possible" as described in the definition of PMP (e.g. HMR 57 Section 1.2). In other words, various levels of conservatism and/or extreme aspects of storms that would not occur in a PMP storm environment should not be compounded together to generate unrealistic results in either the PMP values or the hydrologic applications of those values.

Applied Weather Associates' (AWA) PMP derivation process is based on analysis of historic storms (Tomlinson et al 2010) and therefore incorporates a storm based approach, following many of the methods and assumptions used to develop the set of HMR documents produced by the NWS. The storm search and selection of storms analyzed emphasized storms with the largest rainfall values that occurred over areas that are both meteorologically and topographically similar to the basin or region being studied. Each storm type that occurs over the region is analyzed. For this paper, several area sizes ranging from 5 to 500 square miles were analyzed to determine how the temporal distribution of the PMP varies by area size and hence affects the hydrologic model input parameter used to produce the PMF. The storm types for these basin included local convective (thunderstorms), remnant tropical storms, and general frontal storms. For example purposes, the results for the local convective (thunderstorm) storm event are discussed.

## Temporal Distribution of Precipitation

Understanding and quantifying how rainfall associated with the PMP design storm accumulates over a given duration (e.g. 6 -hours or 72 -hours) is essential to the modeling of the PMF. To determine the temporal distribution of the PMP rainfall, storms analyzed using the Storm Precipitation Analysis System (SPAS) (Parzybok and Tomlinson 2006) and used to derive PMP for the example study basin were evaluated. Storms were grouped by type; local, tropical, and general. Local storms were analyzed from 1-6 hours. Tropical and general storms were analyzed on an hourly basis from 1 through 72 hours.

## Building the Areal Timing Database

A database of observed areal precipitation depths and corresponding area sizes for each storm was the first step in the development of the areal timing procedure. SPAS has analyzed 42 storms as part of the development of the storm database used in the derivation of an updated PMP for the state of Arizona to replace HMR 49 (Hansen et al 1977). One product
of the SPAS storm analysis is a gridded representation of the storm rainfall (at approximately $1 / 3^{\text {rd }}$ of a square mile) at hourly temporal resolution. When NEXRAD (Next Generation Weather Radar) is available and incorporated into the SPAS analysis, this temporal resolution is refined to 5 -minute intervals.

The gridded rainfall data were used to identify the grid cell with the highest total storm rainfall for a given duration, this was considered the "storm center". Starting with hour 1 of the storm, software computed the area size and depth of precipitation at the storm center grid cell; often this was zero because the storm had not yet begun. The program then determined the next three largest adjacent (touching) grid cells to the storm center grid cell. Based on these four grid cells, the average areal precipitation and area size was computed and saved. To expedite processing speed, if all of the grid cells were zero, the software accepted all of the adjacent grid cells. Then, the next highest three adjacent cells were added and area and precipitation statistics computed. The process of adding the next highest three adjacent grid cells was repeated until a maximum desired area size was reached; 500 sqmi was the maximum for local storms and 1000sqmi for the general and tropical storms. Figure 1.shows a sequence of images depicting the areally constrained precipitation associated with hour 6 of SPAS Storm 1085-Wenden/Bouse 2008. The entire process was repeated for all subsequent hours of the storm.


Figure 1. An illustrative sequence of images depicting the areally constrained precipitation associated with hour 6 of SPAS Storm 1085-Yellow is no data.

The constraint of only including adjacent (touching) grid cells helped to isolate only the precipitation associated with the "storm" or associated clusters of "storms." However, when
the storm center was relatively dry for a specific hour, the process of adding the next heaviest grid cells often caused the analysis area to expand outward to wetter grid cells some distance from the "storm center" at larger area sizes. This behavior did not adversely affect the results since these particular hours of the storm did not represent the heaviest precipitation at the "storm center" and therefore were not used in the final results.

The result of this step was a complete database of area sizes and corresponding average precipitation depths for each hour of the storm. Table 2.0 displays sample output from SPAS Storm 1085-Wenden/Bouse 2008. Notice the areally average rainfall (avgppt_in) decreases with increasing area size, as you one would expect.

Table 1. A portion of the depth-area database for SPAS Storm 1085 Wenden/Bouse 2008 08/25/2008-8/27/2008 in/around La Paz County, Arizona.

| All area size summary |  |  |  |
| :---: | :---: | :---: | :---: |
| Storm number: 1085 |  |  |  |
| Latitude (dd): 33.915 |  |  |  |
| Longitude (dd): -113.905 |  |  |  |
| hr, | nocells, | avgppt_i | area_sqmi |
| 5, | 1, | 1.507, | 0.396 |
| 5, | 5, | 1.495, | 1.584 |
| 5, | 9, | 1.432, | 2.772 |
| 5, | 13, | 1.416, | 3.960 |
| 5, | 17, | 1.412, | 5.147 |
| 5, | 21, | 1.390, | 6.335 |
| 5, | 25, | 1.354 , | 7.523 |
| 5, | 29, | 1.321, | 8.711 |
| 5, | 33, | 1.287, | 9.898 |
| 5, | 37, | 1.253, | 11.086 |
| 5 , | 41, | 1.220, | 12.274 |
| 5, | 45, | 1.189, | 13.462 |
| 5, | 49, | 1.155, | 14.650 |
| 5, | 53, | 1.122, | 15.837 |
| 5, | 57, | 1.088, | 17.025 |
| 5, | 61, | 1.054, | 18.213 |
| 5, | 65, | 1.022, | 19.401 |
| 5, | 69, | 0.990, | 20.589 |

## Areal Timing Computations and Results

The second step in the areal timing analysis used the results from the first step, to determine the temporal distributions of precipitation. Scripts were written to determine the maximum precipitation accumulations for the duration of interest (i.e. local storms 6-hr and tropical and general storms $24-\mathrm{hr}$ and $72-\mathrm{hr}$ ) using a moving window. The accumulations were converted into a ratio of the cumulative precipitation to the maximum accumulated precipitation for that duration, and a ratio of the cumulative time to the total time. The summation of the ratios always had a value of $100 \%$.

To accomplish this, the script was used to calculate average precipitation for set of area sizes (i.e. 1-,5-,10-,25-,50-,75-,100-,110-,250-,500-sqmi). Linear interpolations of precipitation from the two bounding area sizes were used to calculate the rainfall at standardized area sizes. A grouped file was created that contained the hour of precipitation, the standard area size, and the average precipitation for each hour of the storm analysis. An example for hour 10 is shown in Table 2.

Table 2. Example of standardize area and the associated average precipitation.

| Hour | Area | Avg. Ppt |
| :---: | :---: | :---: |
| 10 | 1 | 1.67 |
| 10 | 5 | 1.49 |
| 10 | 10 | 1.29 |
| 10 | 25 | 0.83 |
| 10 | 50 | 0.59 |
| 10 | 100 | 0.42 |

From the grouped file created above (Table 2.), a subset of data for the entire storm analysis duration, were created for each area size. A subset for $100-\mathrm{mi}^{2}$ are shown in Table 3.

Table 3. Example of 100-mi2 average precipitation for a subset of storm duration.

| Hour | Area | Avg. Ppt |
| :---: | :---: | :---: |
| 1 | 100 | 0 |
| - | 100 | - |
| 15 | 100 | 0.24 |
| 16 | 100 | 1.43 |
| 17 | 100 | 1.26 |
| 18 | 100 | 0.33 |
| 19 | 100 | 0.33 |
| 20 | 100 | 0 |

A different script (Areal_Pct_Timing.R ) was used with the areal files (Table 2.) to determine the areal timing. The script calculated an accumulation using a moving window (based on durations of 6-, 24- or 72-hours), the maximum areal accumulation is determined for the duration of interest. The first hour in the accumulation needed to have precipitation, if it did not the window was adjusted until the first hour had precipitation. The percent of maximum accumulation was then calculated and hourly precipitation was divided by the maximum accumulated areal precipitation. This calculated the percent accumulation for the duration (0 to 1 ) and percent of duration (0 to 1).

Areal files for each standard area size were created. Each file included area size, incremental rainfall, accumulation rainfall, percent, percent accumulation, and percent duration. An example of a $6-\mathrm{hr} 50-\mathrm{mi}^{2}$ areal precipitation timing file is shown in Table 4.

Table 4. Example of $6-\mathrm{hr} 50-\mathrm{mi}^{2}$ areal precipitation timing.

| Area | Precipitation | Accumulation | Percent | Percent <br> Accumulation | Percent <br> Duration |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 50 | 0.51 | 0.51 | 0.134 | 0.134 | 0.1667 |
| 50 | 1.06 | 1.57 | 0.277 | 0.411 | 0.3333 |
| 50 | 0.22 | 1.79 | 0.058 | 0.469 | 0.5 |
| 50 | 0.3 | 2.09 | 0.079 | 0.547 | 0.6667 |
| 50 | 0.29 | 2.38 | 0.076 | 0.623 | 0.8333 |
| 50 | 1.44 | 3.82 | 0.377 | 1 | 1 |

A total of nine NEXRAD SPAS storms and three non-NEXRAD SPAS storms were used for this portion of the temporal distribution analysis. The location of the storm center, for each storm analysis, was used for the temporal distribution calculations. From this a set of curves was derived for each local storm for 1 through 6 hours at the 75 -square mile area size (most relevant for the example study basin). This set of curves was then analyzed by a hydrologist to determine which produced the most hydrologically stressing storm pattern (i.e. front loaded, middle loaded, back loaded). These values were then used to derive the sequential timing which represents the most stressing and physically possible result for local storms at the 1 through 6 -hour time period (Table 5.).

Table 5. Timing of 1 through 6 -hour local storm data in 1 -hour sequential increments.

Sequential Hourly Timing of a Front Loaded Local Storm

| 1-hr | 2-hr | 3-hr | 4-hr | 5-hr | 6-hr |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $5 \%$ | $45 \%$ | $82 \%$ | $96 \%$ | $100 \%$ | $100 \%$ |

## Derivation of the Temporal Distribution of PMP for Local Storms Using the Results of the Areal Timing

The above timing values were then applied to the local storm PMP values. In order to accomplish this both the temporal distribution of the PMP values and the areal dependent timing values were analyzed and combined to derive the areal timing distribution of the PMP values specific to the basin being analyzed for 1 through 6 -hours at the 75 square mile are size.

In this example, the PMP values derived for the study basin were analyzed from a set of 28 storm events. However, because the final PMP values are a combination of those storms ${ }^{1}$ the temporal distribution of those PMP values is not readily apparent from those data alone. Therefore, it was necessary to derive a PMP temporal distribution for use in applying the PMP data in hydrological applications, specifically for the derivation of the PMF. To accomplish this, AWA followed a similar approach used to derive the PMP values, i.e. use a storm based approach to derive the answer. In this case, each of the 28 storms which were analyzed using

[^0]the SPAS analysis(either with NEXRAD or without) were evaluated using our areal timing software as described above to determine the variation in temporal distribution of those events as the area sizes changed.

## Derivation of the Temporal Distribution

The procedure used in the development of PMP values includes the use of DepthDuration (DD) curves (Figure 2). For each area size analyzed, a curve is developed that provides rainfall values for the durations analyzed at the temporal increment used in the analysis. For local storms, hourly increments were analyzed for 1-hour to 6-hour durations. The construction of the DD curves is such that the 1-hour depth has the largest rainfall value for a given area size for any 1 hour period, the 2 -hour has the largest rainfall value for the 2 hour period, etc. Using these curves, changes in the rainfall values between adjacent durations can be computed, e.g. 1-hour to 2-hour, 2-hour to 3-hour, etc.

Depth-Duration Chart of Enveloped StormData for
the Example Basin


Figure 2. Example Depth-Duration chart for 1 through 6-hours and from 1 square mile through 5000 square miles.

Unfortunately, the NEXRAD storms are only a subset of the total number of storms used to derive PMP in any study. Therefore, SPAS NEXRAD storms only partially influenced the final DD curves. Consequently, the areally dependent timing curves are based on fewer storms than the final DD curves used to derive PMP.

It is important to recognize what significant information is provided by each analysis. The DD curves provide the maximum rainfall that occurred during various durations at any point during the storm. The maximum 1-hour value is provided along with maximum values for other durations. Hence, the percent of the total rainfall can be calculated for each incremental value. Beginning with the 1-hour value, its percent of the total storm value can be computed. Then the difference between the 2 -hour value and the 1 -hour value can be determined and its percent of the total storm value can be computed. Next the difference between the 3 -hour value and the 2 -hour value can be determined and its percent of the total storm value can be computed. This procedure continues until the percent of rainfall is computed for each hourly increment of the DD curve. The first 1-hour increment from the DD curve provides the largest hourly rainfall and hence the largest percentage. The second 1-hour increment provides the next largest hourly rainfall and hence the second largest percentage, etc. The percent of total storm rainfall is computed for each hour from the largest hourly rainfall amount to the smallest hourly rainfall amount (Table 6.).

Table 6. Increments of PMP rainfall values for 1-6 hours based on DD curve and areal timing analysis.

|  | 1-hr | 2-hr | 3-hr | 4-hr | 5-hr | 6-hr |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| DD curve rainfall values | 3.7" | 5.8" | 7.2" | 8.4" | 9.0" | 9.0" |
| DD curve \% | 41\% | 23\% | 16\% | 13\% | 7\% | 0\% |
| Timing curve \% | 5\% | 45\% | 82\% | 96\% | 100\% | 100\% |
| Timing curve increment \% | 5\% | 40\% | 37\% | 14\% | 4\% | 0\% |
| Timing curve increment order | 4 | 1 | 2 | 3 | 5 | 6 |
| Adjusted timing curve increment \% | 13\% | 41\% | 23\% | 16\% | 7\% | 0\% |
| Adjusted timing curve accumulated \% | 13\% | 54\% | 77\% | 93\% | 100\% | 100\% |
| Timed rainfall values | 1.2" | 4.9" | 6.9" | 8.4" | 9.0" | 9.0" |

## Areal Timing Curves

Timing curves were constructed as a function of area size for local storms analyzed with NEXRAD data that were part of the example study ( 5 storms). These curves were constructed using individual storms and the results can be compared to one another to establish a composite areal timing curve for each area size.

The series of curves developed varied from large gradients of rainfall as a percent of total for small area sizes to gentler gradients for larger area sizes. The family of curves can be loosely grouped with area sizes less than 25-square miles being similar, area sizes of 50- and 100-square miles more separated transitioning between small and large area sizes, with 250and 500-square miles curves grouped in a similar fashion (Figure3). Notice how much the 75square mile area size curve varies from the $5-50$-square mile curves from hours 2-5. This could result in substantial difference in the PMP values and how the PMF is calculated for a basin versus simply using one areal timing distribution regardless of the area size of the basin being studied.

The areally dependent timing curves provide the sequence of the rainfall as it occurred during the storm. For each hour of the storm, the percent of the total rainfall that occurred
during that hour were computed. From these values, the ordering of the heaviest rainfall as it occurred during the storm is provided. For example, if the highest percentage occurred during the third hour, then the largest rainfall occurred during that hour. If the next highest percentage occurred during the second hour, then the second largest rainfall occurred during that hour, etc.


Figure 3. Family of timing curves for local convective thunderstorms for 5 SPAS NEXRAD storm analyses in Arizona. 75-square mile area size is highlighted in red.

## Recommended Temporal Distribution of the Local Storm PMP

Based on the combination of the above discussed results, a storm based temporal distribution of the local storm PMP was derived. Because different storms are used in each analysis, i.e. DD curve construction and areal curve timing construction, the hourly percentages were not the same. However, each curve had information about hourly rainfall, the DD curves provided the percent of total storm rainfall for the largest hourly rainfall and the timing curves provided the hour during the storm when the largest hourly rainfall occurred. For a given area size, by using ordering from the timing curve and the percent of storm total PMP rainfall from the DD curve, a sequence of hourly rainfall percentages can be determined that maintains ordering consistency with the timing curve and percentage consistency with the DD curve (Table 7.).

Below is an example for local convective storms using the timing curve for the 75square mile area size. The total storm duration of 6 hours is used with a time increment of 1 hour.

Using this procedure, the PMP values from the DD curves are preserved for incremental rainfall values and the order of timing is preserved from the timing curve.

Table 7. Temporal distribution of the local storm PMP rainfall.

|  | 1-hr | 2-hr | 3-hr | 4-hr | 5-hr | 6-hr |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Areal Timing Curve <br> Incremental \% of Total <br> PMP Rainfall | $13 \%$ | $41 \%$ | $23 \%$ | $16 \%$ | $7 \%$ | $0 \%$ |
| Areal Timing Curve <br> Accumulated \% of Total <br> PMP Rainfall | $13 \%$ | $54 \%$ | $77 \%$ | $93 \%$ | $100 \%$ | $100 \%$ |
| Timed Rainfall Values <br> Using PMP Values | $1.2^{\prime \prime}$ | $4.9^{\prime \prime}$ | $6.9^{\prime \prime}$ | $8.0^{\prime \prime}$ | $9.0^{\prime \prime}$ | $9.0^{\prime \prime}$ |

## Conclusions

The results of this analysis show how important it is to properly quantify the areal timing variations of analyzed storms and PMP rainfall before applying those values in a hydrologic analysis to derive the PMF. In the past, this was not possible, as the spatial variations of rainfall could not be analyzed and quantified accurately over varying temporal scales and varying area sizes. However, the ability to analyze storms with SPAS NEXRAD has provided the capability to overcome this for the first time. These results provide valuable information in understanding how rainfall varies in time as areal size increases for a storm domain, providing a more refined input parameter for the hydrologic analysis of rainfall.

## References

1. Corrigan, P., Fenn, D.D., Kluck, D.R., and J.L. Vogel, 1999: Probable Maximum Precipitation Estimates for California. Hydrometeorological Report No. 59, U.S. National Weather Service, National Oceanic and Atmospheric Administration, U.S. Department of Commerce, Silver Spring, MD, 392 pp.
2. Hansen, E.M., Schwarz, F.K., and J.T. Riedel, 1977: Probable Maximum Precipitation Estimates, Colorado River and Great Basin Drainages. Hydrometeorological Report No. 49, National Weather Service, National Oceanic and Atmospheric Administration, U.S. Department of Commerce, Silver Spring, MD, 161 pp.
3. Colorado River and Great Basin Drainages. Hydrometeorological Report No. 50, National Weather Service, National Oceanic and Atmospheric Administration, U.S. Department of Commerce, Silver Spring, MD, 167 pp.
4. $\qquad$ Fenn, D.D., Schreiner, L.C., Stodt, R.W., and J.F., Miller, 1988: Probable Maximum Precipitation Estimates, United States between the Continental Divide and the $103^{\text {rd }}$ Meridian, Hydrometeorological Report Number 55A, National weather Service, National Oceanic and Atmospheric Association, U.S. Dept of Commerce, Silver Spring, MD, 242 pp.
5. $\qquad$ Schwarz, F.K., and J.T. Riedel, 1994: Probable Maximum PrecipitationPacific Northwest States, Columbia River (Including portion of Canada), Snake River, and Pacific Drainages. Hydrometeorological Report No. 57, National Weather Service, National Oceanic and Atmospheric Administration, U.S. Department of Commerce, Silver Spring, MD, 353 pp.
6. Parzybok, T.W., and E. M. Tomlinson, 2006: A New System for Analyzing Precipitation from Storms, Hydro Review, Vol. XXV, No. 3, 58-65.
7. Tomlinson, E.M., Kappel W.D., Parzybok, T.W., Hultstrand, D., Muhlestein, G., and B. Rappolt, May 2008: Site-Specific Probable Maximum Precipitation (PMP) Study for the Wanahoo Drainage Basin, Prepared for Olsson Associates, Omaha, Nebraska.
8. $\qquad$ Kappel W.D., Parzybok, T.W., Hultstrand, D., Muhlestein, G., and B. Rappolt, June 2008: Site-Specific Probable Maximum Precipitation (PMP) Study for the Blenheim Gilboa Drainage Basin, Prepared for New York Power Authority, White Plains, NY.
9. $\qquad$ Kappel W.D., and T.W. Parzybok, February 2008: Site-Specific Probable Maximum Precipitation (PMP) Study for the Magma FRS Drainage Basin, Prepared for AMEC, Tucson, Arizona.
10. $\qquad$ Kappel W.D., Parzybok, T.W., Hultstrand, D., Muhlestein, G., and P. Sutter, December 2008: Statewide Probable Maximum Precipitation (PMP) Study for the state of Nebraska, Prepared for Nebraska Dam Safety, Omaha, Nebraska.
11. $\qquad$ Kappel, W.D., and Tye W. Parzybok, February 2009: Site-Specific Probable Maximum Precipitation (PMP) Study for the Tuxedo Lake Drainage Basin, New York.
12. $\qquad$ Kappel, W.D., and Tye W. Parzybok, July 2009: Site-Specific Probable Maximum Precipitation (PMP) Study for the Scoggins Dam Drainage Basin, Oregon. 13. $\qquad$ E. M., and W.D. Kappel, October 2009: Revisiting PMPs, Hydro Review, Vol. 28, No. 7, 10-17.
13. $\qquad$ , Kappel, W.D., and Tye W. Parzybok, February 2010: Site-Specific Probable Maximum Precipitation (PMP) Study for the Magma FRS Drainage Basin, Arizona.
14. US Army Corps of Engineers Storm Studies, 1973, Storm Rainfall in the United States Depth-Area-Duration Data
15. World Meteorological Organization, 1986: Manual for Estimation of Probable Maximum Precipitation.

[^0]:    ${ }^{1}$ Following standard PMP derivation procedures, envelopment of the largest storms at each area size and duration is used to determine PMP values for a basin

